

Influence of Gravity on noncommutative Dirac equation

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February 1, 2008

Abstract

In this paper, we investigate the influence of gravity and noncommutativity on Dirac equation. By adopting the tetrad formalism, we show that the modified Dirac equation keeps the same form. The only modification is in the expression of the covariant derivative. The new form of this derivative is the product of its counterpart given in curved space-time with an operator which depends on the noncommutative θ -parameter. As an application, we have computed the density number of the created particles in presence of constant strong electric field in an anisotropic Bianchi universe.

1 Introduction

Recently there has been a large interest in the study of noncommutative field theory. Taking space-time coordinates to be noncommutative is an old idea dates back to the work of Snyder[1]. The goal was that the introduction of a noncommutative structure to space-time at small length scales could introduce an effective cut-off which regularize divergences in quantum field theory. However this theory was plagued with several problems such as the violation of unitarity, causality, etc.. which make people abandon it. Recently, the appearance of such theories as certain limits of string theory, D-brane and M-theory have generated a revival of interest for field theory on a noncommutative space-time[2, 3]. For a review of noncommutative field theories see[4].

The noncommutative space-time is characterized by operators \hat{x}_μ satisfying the relation

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} = \frac{i}{\Lambda_{NC}^2} C^{\mu\nu} \quad (1)$$

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where $\theta^{\mu\nu}$ is antisymmetric matrix which has the dimension of area. Its role can be compared to that of the Planck constant \hbar which quantifies in quantum mechanics the level of noncommutativity between space and momentum. $C^{\mu\nu}$ are dimensionless parameters which are presumably of order unity, and Λ_{NC} is the scale energy where the noncommutative effects of the space-time become relevant. At the beginning this scale was expected to manifest around the Planck scale which makes the noncommutative effects of space-time out of reach of the current colliders. However if we assume the possibility of large extra dimensions, it is likely that Λ_{NC} could be set at a TEV scale[5].

There are two different settings for noncommutative field theories. The first one is based on the so-called Moyal or star product. The other setting uses the Sieberg-Witten map[3, 4, 5, 6]. In this work we shall adopt the first approach where field theory in a noncommutative space-time can be described as the ordinary field theory where every product of fields is replaced with the Moyal or star product $(*)$ defined as

$$f(x) * g(x) = [\exp(\frac{i}{2}\theta^{\mu\nu}\partial_\mu^{(\xi)}\partial_\nu^{(\eta)})f(x+\xi)g(x+\eta)]_{\xi=\eta=0} \quad (2)$$

where f and g are two arbitrary differentiable fields. This deviation from the standard theory causes the violation of Lorentz invariance when θ is considered as a constant matrix, except if this matrix is promoted to a tensor related to the contracted Snyder's Lie algebra[7, 8]. Also, it has been shown that the problem of unitarity appears in the study of quantum theories in flat space with time-space noncommutativities ($\theta^{0i} \neq 0$)[9, 10]. Most applications of ordinary field theories have been reconsidered in the context of noncommutative geometry [11, 12, 13, 14]. Among these applications, the pair production in the presence of electric field has been treated by a nonperturbative theory with a noncommutative Dirac equation in the absence of gravity[15]. The second section is devoted to establishing the modified Dirac equation in curved space-time. This modification is due to the influence of noncommutativity and gravity on Dirac particle. In fact, we will adopt the tetrad formalism[16] as a tool to formulate the final equation up to the first order in the noncommutative parameter θ . As an application, we will propose in the third section an example of pair production in the presence of strong electric field in the Bianchi 1 universe.

2 Modified Dirac Equation in curved space

It is well known that to determine the effects of gravitation on general dynamical system, one has just to take the special-relativistic motion equations in the absence of gravitation and replace all components of the Lorentz tensors with its components given in non flat space. Also, we replace all derivatives ∂_α with covariant derivatives and the Minkowski metric $\eta_{\alpha\beta}$ with $g_{\alpha\beta}$. It is possible to determine the effects of gravity on spinor field by using the tetrad formalism[16] which is based on the principle of equivalence. The metric in any general non

inertial coordinate system is given by

$$g_{\mu\nu}(x) = e_{\mu}^{(\alpha)} e_{\nu}^{(\beta)}(x) \eta_{\alpha\beta} \quad \text{and} \quad e_{\mu}^{(\alpha)} = \partial_{\mu} \zeta_x^{(\alpha)} \quad (3)$$

$e_{\mu}^{(\alpha)}(x)$ are the vierbeins or tetrads that connect between the curved space and its local flat counterpart. Its inverse is $e_{(\alpha)}^{\mu} = \frac{\partial x^{\mu}}{\partial \zeta_x^{\alpha}}$ such that

$$e_{(\alpha)}^{\mu} e_{\nu}^{(\alpha)} = \delta_{\nu}^{\mu} \quad (4)$$

We can use the vierbeins to refer the components of contravariant vector $T^{\mu}(x)$ into the local coordinate system ξ_x^{α}

$$T^{\mu} \rightarrow e_{\alpha}^{(\mu)} T^{\alpha} \quad (5)$$

Generally, for any tensor T_{β}^{α} we have

$$T_{\beta}^{\alpha} \rightarrow e_{\mu}^{(\alpha)} e_{(\beta)}^{\nu} T_{\nu}^{\mu} \quad (6)$$

Now, we consider that the flat space time is deformed by θ -parameter which makes their coordinates do not commute given by (1) and we take $\theta^{\alpha\beta} = \theta \epsilon^{\alpha\beta}$ ($\epsilon^{\alpha\beta}$ is Levi-Cevita tensor).

To study the influence of gravity on Dirac equation given in a deformed space time (1), we start in the first step by giving the noncommutative Dirac equation that describes spinorial particle in the presence of electromagnetic field and the absence of gravity[15]

$$[\gamma^{\mu}(\partial_{\mu} - ieA_{\mu}(\hat{x})) + m]\psi(\hat{x}) = 0, \quad \text{with} \quad [\hat{x}^{\mu}, \hat{x}^{\nu}] = i\theta^{\mu\nu}, \quad (7)$$

where γ^{μ} are the usual Dirac matrices that satisfy $[\gamma^{\mu}, \gamma^{\nu}] = 2\eta^{\mu\nu}$. Choosing

$$\begin{aligned} \gamma^0 &= \begin{pmatrix} -i\sigma_1 & 0 \\ 0 & i\sigma_2 \end{pmatrix}, \gamma^1 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \\ \gamma^2 &= \begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix}, \gamma^3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix} \end{aligned} \quad (8)$$

σ_1, σ_2 and σ_3 are Pauli matrices.

By using the multiplication law of star product (2) up to the first order into eq. (7), we get

$$[\gamma^{\mu}(\partial_{\mu} - ieA_{\mu} + \frac{e}{2}\theta^{\alpha\rho}(\partial_{\alpha}A_{\mu})\partial_{\rho}) + m]\psi = 0 \quad (9)$$

due to the smallness of the θ -parameter ($\theta \ll 1$). As a consequence, the eq. (7) has an analogical equation in the commutative space

$$[\gamma^{\mu}(D_{\mu}(x) - ieA_{\mu}(x)) + m]\psi = 0, \quad \text{with} \quad [x^{\mu}, x^{\nu}] = 0, \quad (10)$$

where D_{μ} is the modified derivative

$$D_{\mu} = M_{\mu}^{\rho}(\theta)\partial_{\rho} \quad \text{with} \quad M_{\mu}^{\rho}(\theta) = \delta_{\mu}^{\rho} + \frac{e}{2}\theta^{\alpha\rho}(\partial_{\alpha}A_{\mu}). \quad (11)$$

The part which does not contain the θ -dependent corrections and corresponds to the usual Dirac operator remains covariant.

Then, similar to the commutative case, and in order to establish the modified Dirac equation in the presence of gravity, we follow the tetrad formalism approach and make the following changes in eq.(10)[16]

$$\begin{aligned} A_\mu &\rightarrow e_{(\mu)}^\alpha A_\alpha, \\ \partial_\alpha A_\mu &\rightarrow e_{(\alpha)}^\nu e_{(\mu)}^\alpha (\partial_\nu A_\alpha), \\ \partial_\mu \psi &\rightarrow e_{(\mu)}^\alpha (\partial_\alpha - \Gamma_\alpha) \psi, \end{aligned} \quad (12)$$

where Γ_μ is the spin connection, its role is to conserve the covariance that exist already in the commutative part (does not depend on matrix θ) in order not to make a conflict in the classical limit when $(\theta \rightarrow 0)$

$$\Gamma_\mu = \frac{1}{4} g_{\lambda\alpha} [(\partial_\mu e_\nu^{(\beta)}) e_{(\beta)}^\alpha - \Gamma_{\nu\mu}^\alpha] s^{\lambda\nu} \quad (13)$$

with

$$s^{\lambda\nu} = \frac{1}{2} e_{(\rho)}^\mu e_{(\sigma)}^\nu [\gamma^\rho \gamma^\sigma - \gamma^\sigma \gamma^\rho] \quad (14)$$

and $\Gamma_{\nu\mu}^\alpha$ is the affine connection, which is written in function of the metric $g_{\lambda\alpha}$ as

$$\Gamma_{\nu\mu}^\alpha(x) = (1/2) g^{\alpha\beta} (\partial_\nu g_{\mu\beta} + \partial_\mu g_{\beta\nu} - \partial_\beta g_{\mu\nu}). \quad (15)$$

Finally, by substituting the changes given in eqs. (12) into the modified derivative (11), we get

$$D_\mu \rightarrow e_{(\mu)}^\alpha \hat{D}_\alpha \quad (16)$$

where \hat{D}_α is the modified Dirac derivative in the presence of gravity

$$\hat{D}_\mu = \hat{M}_\mu^\rho(\theta) (\partial_\rho - \Gamma_\rho), \quad (17)$$

with

$$\hat{M}_\mu^\rho(\theta) = \delta_\mu^\rho + \frac{e}{2} \theta^{\alpha\rho} e_{(\alpha)}^a e_{(\mu)}^b (\partial_a A_b) \quad (18)$$

the modified Dirac equation in the presence of gravity is

$$[\tilde{\gamma}^\alpha (\hat{D}_\alpha - ie A_\alpha) + m] \psi = 0. \quad (19)$$

The matrices $e_{(\nu)}^\mu, e_\mu^{(\nu)}$ connect between the $\tilde{\gamma}^\mu$ and Minkowski γ^μ Dirac matrices as follow:

$$\tilde{\gamma}^\alpha = \gamma^\mu e_{(\mu)}^\alpha \quad (20)$$

$\tilde{\gamma}^\alpha$ satisfy the anticommutation relation

$$[\tilde{\gamma}^\alpha, \tilde{\gamma}^\beta]_+ = 2g^{\alpha\beta}. \quad (21)$$

3 Creation of Dirac particles in the presence of a constant electric field in an anisotropic Bianchi 1 universe

The creation of particles induced by the vacuum instability is among the interesting non perturbative phenomena which can be realized with a presence of strong electromagnetic field.

In this section, we will see an example of Dirac particles created in a cosmological anisotropic Bianchi 1 universe in the presence of a strong constant electric field using the modified Dirac equation (19), in order to deduce the effect of the noncommutativity on the density of created particles.

The line element that defines the Bianchi universe is given by

$$ds^2 = -dt^2 + t^2(dx^2 + dy^2) + dz^2, \quad (22)$$

then the metric

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & t^2 & 0 & 0 \\ 0 & 0 & t^2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (23)$$

and its inverse is

$$g^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{1}{t^2} & 0 & 0 \\ 0 & 0 & \frac{1}{t^2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (24)$$

Since the line element is diagonal, we choose to work in the diagonal tetrad

$$e_{(\alpha)}^\mu = \sqrt{|g^{\mu\beta}|} \delta_{\alpha\beta}, \quad (25)$$

from (13), (22) and (24), we get all components of spin connection

$$\Gamma_1 = \frac{1}{2}\gamma^0\gamma^1, \Gamma_2 = \frac{1}{2}\gamma^0\gamma^2, \Gamma_3 = \Gamma_0 = 0 \quad (26)$$

We fix the third axis of our frame parallel to the direction of constant electric field \vec{E} . This means that the system possesses a rotational symmetry along z axis that preserves the invariance of line element given by (22). In such cases, the electric field has only one component $E_z = E$ which corresponds to the chosen potential

$$A_\mu = (0, 0, 0, -Et). \quad (27)$$

The modified Dirac equation (19) in anisotropic Bianchi universe (22) with the presence of electric field represented by the potential A_μ in (27) is

$$[\gamma^\alpha e_{(\alpha)}^\mu ((\delta_\mu^\rho + \frac{e}{2}\theta^{0\rho}\delta_\mu^3 E)(\partial_\rho - \Gamma_\rho) - ieA_\mu) + m]\psi = 0 \quad (28)$$

To visualize the influence of the noncommutativity on the creation of Dirac particles, we use the parametrization that determines the elements of θ -Matrix from the direction of background electromagnetic fields[11]

$$C^{\mu\nu} = \begin{pmatrix} 0 & \sin \alpha \cos \beta & \sin \alpha \sin \beta & \cos \alpha \\ -\sin \alpha \cos \beta & 0 & \cos \gamma & -\sin \gamma \sin \beta \\ -\sin \alpha \sin \beta & -\cos \gamma & 0 & -\sin \gamma \cos \beta \\ -\cos \alpha & \sin \gamma \sin \beta & \sin \gamma \cos \beta & 0 \end{pmatrix}$$

and again keep the background electric field parallel to z axis to benefit from the existed rotational symmetry in order to be sure that the density number of particles remains unchanged under this symmetry. Although, the Lorentz symmetry is generally broken when θ -matrix is not considered as a tensor and all its elements are identical in all reference frames. Geometrically, we have $\alpha = 0$ to obtain $\theta^{0i} = \theta^{03} = \theta$, and set all the remaining time space components equal to zero), then the eq (28) becomes

$$[\gamma^0 \frac{\partial}{\partial t} + \gamma^3 (e^{\frac{\epsilon}{2}\theta E} \frac{\partial}{\partial z} + ieEt) + \frac{1}{t} (\gamma^1 \frac{\partial}{\partial x} + \gamma^2 \frac{\partial}{\partial y}) + m] \hat{\psi} = 0. \quad (29)$$

We have used $\hat{\psi} = t\psi$ to cancel the term related to the spin connection in the equation (28), and have taken

$$e^{\frac{\epsilon}{2}\theta E} \approx (1 + \frac{e}{2}\theta E). \quad (30)$$

Since the operators $(\partial_x, \partial_y, \partial_z)$ commute with the operator acting on $\hat{\psi}$, the general solution is

$$\hat{\psi} = \exp(\vec{k} \cdot \vec{r}) \begin{pmatrix} \varphi(t) \\ \chi(t) \end{pmatrix} \quad (31)$$

where $\varphi(t), \chi(t)$ are the vectors which depend only on time.

$$\vec{r} = (x, y, z) \quad \text{and} \quad \vec{k} = (ik_x, ik_y, ik_z). \quad (32)$$

The operator acting on $\Psi = \gamma^3 \gamma^0 \hat{\psi}$ can be written as the sum of two commuting operators \hat{O}_1 and \hat{O}_2

$$[\hat{O}_1, \hat{O}_2] = 0, \quad (33)$$

such that

$$\frac{\hat{O}_1}{t} = [\gamma^3 \frac{\partial}{\partial t} + i\gamma^0 (e^{\frac{\epsilon}{2}\theta E} k_z + eEt) + \gamma^3 \gamma^0 m] \quad (34)$$

$$\hat{O}_1 = i(\gamma^1 k_x + \gamma^2 k_y) \gamma^3 \gamma^0 \quad (35)$$

By considering Ψ as an eigenvector of \hat{O}_2 with eigenvalues k , we get

$$\hat{O}_2 = \begin{pmatrix} \varphi(t) \\ \chi(t) \end{pmatrix} = k \begin{pmatrix} \varphi(t) \\ \frac{k_x \sigma_x}{ik_y - k} \varphi(t) \end{pmatrix} \quad (36)$$

$$k = i\sqrt{k_x^2 + k_y^2} = ik_\perp, \quad (37)$$

and

$$\varphi(t) = \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}. \quad (38)$$

On the other hand, we have Ψ is an eigenvector of \hat{O}_1 , then $f_1(t)$ and $f_2(t)$ satisfy the following differential equations system

$$\frac{\partial f_1(t)}{\partial t} + \frac{k}{t}f_1(t) + (eEt + \tilde{k}_z + im)f_2(t) = 0 \quad (39)$$

$$-\frac{\partial f_2(t)}{\partial t} + \frac{k}{t}f_2(t) + (eEt + \tilde{k}_z - im)f_1(t) = 0 \quad (40)$$

where the modified vector wave

$$\tilde{k}_z = k_z e^{\frac{2}{3}\theta E} \quad (41)$$

Physically, the created particles undergo an added acceleration to their impulses along the direction of background electric field.

In order to determine the density number of created particles, we should take into account the asymptotic behavior of f_1 and f_2 at $t \rightarrow 0$ and $t \rightarrow \infty$.

By considering the ultra relativistic approximation $|E| \gg m, k_z$ and assuming that $|\theta| \ll |\frac{m}{E}|$, while neglecting $|\frac{k_z}{E}|$ and $|\frac{m}{E}|$ in the first order variation of $f_1(t)$ and $f_2(t)$; then eqs. (37), (38) reduce to the same solutions form as that given without the influence of the noncommutativity[17], i.e. we obtain the solutions in terms of Whittaker functions

$$\frac{f_2(t)}{\sqrt{t}} = [C_1 M_{\lambda^+, \mu}(ieEt^2) + C_2 W_{\lambda^+, \mu}(ieEt^2)] \quad (42)$$

$$\frac{f_1(t)}{\sqrt{t}} = [C_3 M_{\lambda^-, \mu+1}(ieEt^2) + C_4 W_{\lambda^-, \mu+1}(ieEt^2)] \quad (43)$$

$$\lambda^\pm = \frac{i(im \pm \tilde{k}_z)^2}{4eE}, \mu = \frac{k}{2} - \frac{1}{2}, \quad (44)$$

where C1,C2 and C3,C4 are arbitrary constants.

We compute the density number of particles creation with the help of the Bogoliubov coefficients α, β [17, 18, 19] which relate between the limit of the negative frequency solutions ψ_0^- at $t \rightarrow 0$ and both ψ_∞^- (the limit negative frequency solution at $t \rightarrow \infty$) and ψ_∞^+ (the limit of the positive frequency solution at $t \rightarrow \infty$), knowing that the negative or positive frequency mode is extracted by using the sign of the operator $i\partial_t$

$$\psi_0^- = \alpha\psi_\infty^- + \beta\psi_\infty^+. \quad (45)$$

By following the same steps given in[17], we deduce the result $(\frac{|\beta|^2}{|\alpha|^2})_{NC}$

$$(\frac{|\beta|^2}{|\alpha|^2})_{NC} = e^{2i\pi\mu} \frac{|\Gamma(\frac{1}{2} + \mu - \lambda^\pm)|^2}{|\Gamma(\frac{1}{2} + \mu + \lambda^\pm)|^2} = e^{-\pi k_\perp} \frac{(\frac{k_\perp}{2} + \tilde{\lambda}) \sinh \pi(\frac{k_\perp}{2} + \tilde{\lambda})}{(\frac{k_\perp}{2} - \tilde{\lambda}) \sinh \pi(\frac{k_\perp}{2} - \tilde{\lambda})} \quad (46)$$

where we have considered that

$$(\frac{k_\perp}{2} + \tilde{\lambda}) = Im(\frac{1}{2} + \mu + \lambda^\pm) \gg Re(\frac{1}{2} + \mu + \lambda^\pm), \quad (47)$$

and

$$\tilde{\lambda} = -\frac{m^2 - \tilde{k}_z^2}{4eE}. \quad (48)$$

The density number of created particles with the influence of noncommutativity is

$$n_{NC} = [(\frac{|\beta|^2}{|\alpha|^2})_{NC}^{-1} + 1]. \quad (49)$$

Remark that the final result of density number of particles creation (49) is independent from the rotational symmetry of space along z axis. This means that our frame has only the particular symmetries and is fixed by the background field. This point of view is coming from the first approach of noncommutative space time[3, 4, 5, 6] based on constant θ -matrix. Finally by using the perturbative expansion of (49) up to the first order of θ -parameter, we get

$$n_{NC} = n + \Delta n \quad (50)$$

n denotes the usual density number of created particles.

Δn denotes the correction due to the noncommutativity such that

$$\Delta n = \frac{\theta k_z^2}{4} e^{-\pi k_\perp} \left[\frac{\sinh(\pi k_\perp)}{\sinh^2(\frac{\pi k_\perp}{2} + \frac{\pi m^2}{4eE})} + \frac{k_\perp}{(\frac{k_\perp}{2} + \frac{m^2}{4eE})^2} \right]. \quad (51)$$

It is clear that when $\theta \rightarrow 0$, we have $\Delta n \rightarrow 0$.

In the absence of the electric field ($E \rightarrow 0$) we can show easily that $\Delta n \rightarrow 0$ and we recover that the density of created particle becomes thermal[20]

$$n_{NC} \approx n \approx e^{-2\pi k_\perp}. \quad (52)$$

4 Conclusion

We have deduced the form of the Dirac equation in the presence of gravity in the framework of a noncommutative space-time.

By adopting the tetrad formalism, we have shown that the modified Dirac equation keeps the standard form except a modification in the expression of the covariant derivative. The new form of this derivative is the product of its counterpart in curved space-time by an operator which depends on the non-commutative θ parameter. As an application, we have computed the density

number of the created particles in presence of constant strong electric field in an anisotropic Bianchi universe. Our calculation shows a correction due to the noncommutativity to be contrasted with the work of Chair et. al. where it was found that there is no correction induced by noncommutativity on a flat space-time. The main perspective of this work is to use θ -matrix as a tensor to preserve the Lorentz covariance in the same spirit of reference[7].

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